AMRIT SCIENCE CAMPUS

B.Sc. CSIT First Semester
Pre-Board Examination - **Set A**

Subject: Mathematics - I | Full Marks: 60
Time: 3 hours | Pass Marks: 24

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group A [$2 \times 10 = 20$].

*Attempt **any two** questions. Each question carries 10 marks.*

1. (a) Using the graph of $y = x^2$, sketch the graph of $y = 3 + (x-2)^2$. [4]
   (b) Sketch the curve represented by $y = xe^x$. [6]
2. (a) Evaluate $\lim\limits_{x \to 0^+} x^x$. [6]
   (b) Does $f(x, y) = \dfrac{xy}{x^2 + y^2}$ have a limit as $(x, y) \longrightarrow (0, 0)$? [4]
3. (a) Using the method of rectangles, evaluate the exact area bounded by X-axis, Y-axis, $x = 1$ and $y = 3x^2$. [5]
   (b) Find the volume of the solid below the surface $z = xy$ and above the rectangle $[0, 6] \times [0, 4]$ using double integrals. [5]

Group B [$8 \times 5 = 40$].

*Attempt **any eight** questions. Each question carries 5 marks.*

4. Use continuity to evaluate $\lim\limits_{x \to 1} \arctan\left(\dfrac{x^2 - 1}{x + 1}\right)$.
5. Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{x^6 + 9}\, dx$, if possible.
6. Discuss the absolute and conditional convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.
7. Using Newton-Raphson Method, find $\sqrt[6]{2}$ correct to eight decimal places.
8. Obtain orthogonal trajectories of the family of circles represented by $x^2 + y^2 = a^2$.
9. If the portion of the curve $y = x^3$ between $x = 0$ and $x = 2$ is rotated about X-axis, then what is the area of the resulting surface of revolution?
10. Find the arc-length of the plane curve represented by $\vec{r}(t) = (\cos^3 t)\vec{i} + (\sin^3 t)\vec{j}$, where $0 \leq t \leq 2\pi$.
11. Find critical points of $f(x, y) = x^2 + y^2 + z^2$ subjected to $x^2 - z^2 = 1$, using Lagrange's multipliers.
12. Find z_x and z_y if $z = z(x, y)$ and is obtained from the equation $x^3 + y^3 + z^3 + 6xyz = 1$.

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Group A [2 × 10 = 20].

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1. (a) Show that Maclaurin's series for $\sin x$ represents $\sin x$ for all $x \in \mathbb{R}$. [4]

   (b) Prove that $f(x,y) = \begin{cases} \ln\left(\dfrac{2x^2 - x^2y^2 + 2y^2}{x^2+y^2}\right) & \text{for } (x,y) \neq (0,0); \\ \ln 2 & \text{for } (x,y) = (0,0) \end{cases}$ is continuous on the Cartesian plane $\mathbb{R}^2$. [6]

2. (a) Prove that the volume of a sphere of radius a is $\frac{4}{3}\pi a^3$. [5]

   (b) Evaluate the double integral of $f(x,y) = x^2 y$ over the triangular region with the vertices at $(0,0), (0,1), (1,0)$. [5]

3. Define Initial Value Problem (IVP) and solve the ODE $y'' + y = x^3 + e^x$ subjected to the conditions $y(0) = 2, y'(0) = 0$. [10]

Group B [8 × 5 = 40].

*Attempt **any eight** questions. Each question carries 5 marks.*

4. Find domain and sketch the graph of $f(x) = \dfrac{3x + |x|}{x}$.
5. Using the formal definition of limits, prove that $(x+3) \to 4$ as $x \to 1$.
6. Sketch the curve of $y = \dfrac{2x^2}{x^2 - 1}$.
7. Verify Rolle's Theorem for $f(x) = x^2$ on $[-1, 1]$.
8. Evaluate $\displaystyle\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2 + r^2}$.
9. Find the arc-length of $y = x^{3/2}$ between $(1,1)$ and $(4,8)$.
10. For what values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{x}$ converge?
11. Find the curvature of the helix $\vec{r}(t) = (\cos t, \sin t, t)$.
12. What is the optimal area of the rectangle inscribed in a 3-4-5 right triangle, provided that its sides are parallel to the legs of the triangle?